NONUNIFORM SAMPLING WITH ADAPTIVE EXPECTANCY BASED ON LOCAL VARIANCE

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Abstract—A new trend in sensor array architectures is to provide compact implementations based on alternative acquisitions and sampling. In particular, with the recent rise of Compressive Sensing (CS), multiple sensing schemes have been developed. However, for the moment, CS reconstruction techniques take a relatively long time to properly converge. Therefore, it limits the sensor resolution and potential applications. On the other hand, it generally involves complex structures and circuitries at the sensor side. This work proposes an acquisition chain performing an adaptive sensing of pseudo-randomly selected samples. This specific nonuniform sampling scheme allows to parallelize and simplify the acquisition thanks to a compact design based on sigma delta converters and cellular automata. Previous works show that compared to state of the art and without an important image degradation, dedicated reconstructions to this specific sampling can considerably reduce the overall computation time.

I. Introduction

Alternative sampling methods and dedicated embedded processing are now investigated to relax on-chip constraints and to improve sensing performances. Indeed, the design of new sensing schemes tends to take advantage of theoretical results related to the Compressive Sensing (CS) paradigm [1] but also of practical results related to adaptive sampling [2]. Indeed, a major interest of developing novel sensing schemes is to find the best compromises between performance of the acquisition system and the relevance of the captured samples for a specific application. The main issue regarding the implementation of a CS-based system is the realisability of the CS measurements [3]. CS aims at compressing the signal directly at the sensing stage allowing a reduction of the amount of information to convert. Alternatively, adaptive sensing has also been investigated to improve the efficiency of sensors. Indeed, by making the sensing adaptive, it allows to select only the most relevant samples or measurements to acquire.

In this paper, we focus on a sensing scheme that combines adaptivity and randomness for sparse sampling. The proposed sensing is based on a random selection of the samples in sub-supports of the signal. Thanks to a sparsity estimation, the expectancy of the number of samples is tuned accordingly for each sub-support. Therefore it is equivalent to an adaptive block-based CS in which the measurement matrix for each block corresponds to the concatenation of randomly selected rows of the spike matrix. It has the advantage to be easily implementable in various systems because of being highly parallelizable in the case of a sensor array for multidimensional signals such as images. Moreover, it can be considered as a nonuniform sampling if the random pattern which is employed to select the samples on each sub-support is different from each other.

Theoretical CS results show that the compression ratio that can be reached for a certain reconstruction accuracy highly depends on the signal sparsity. Indeed, the sparser the signal the more compressible. With respect to the CS theoretical results and regarding the case of Fourier-sparse signals (e.g. images), one of the most appropriate sensing scheme corresponds to a selection of samples at random positions. This consideration is related to the fact that the identity matrix minimizes the coherence with the Fourier basis [1]. When nonuniform sampling [4] is performed, it is relevant to select in priority the samples that carry more information. If the signal is divided into sub-supports, the number of samples to pick up thus have to be in function of the degree of sparsity.

The proposed acquisition chain is well suited for its implementation into an image sensor chip but is not limited to this particular application. It fits with imaging because nonuniform sampling can be tuned according to the block variance which is known to be correlated with the degree of local sparsity [5]. Regarding natural images, Fig. 1 shows that there exists a high correlation between traditional sparsity spaces and the block variance for blocks that are small enough. Being based on the study of a 512 × 512 test images database, those results would slightly differs for a different size of the images. The Pearson coefficient between the variance and four different l1 sparsity estimators is estimated to be higher than 0.6 for block smaller than 32 × 32. Traditional but also alternative regularization objective functions have been tested: Total Variation (TV), Total Variation over 4 axes comprising the diagonals (TV 4axes), Haar wavelet basis and Total Variation in the Daubechies 6 wavelet basis representation.

In digital sensors, one of the most important part is the Analog to Digital Converters (ADCs) that are often considered as bottlenecks. Designed for static signals, our adaptive sampling takes the advantage of the particular structure of a first order Sigma Delta ADC [6] combined with the modularity allowed by a cellular automaton [7]. Indeed, the first order incremental Sigma-Delta is used for both its capability of
averaging a variable input and the easy integration of a digital squaring circuitry. Slight modifications of a traditional cellular automaton with a chaotic behaviour allows the generation of pseudo-random binary vectors with a tunable expectancy at the expense of an output layer of appropriate logical gates. The proposed sampling follows the successive steps:

- Calculation and A/D conversion of the variance of the input signal belonging to the sub-support (performed during a single conversion time).
- Adaptation of reconfiguration of the cellular automaton outputs according to the quantized variance. Update the cellular automaton states (almost instantaneous).
- Scan the sub-support samples and select only the ones corresponding to activated bits of the cellular automaton (last as many conversion times as input discrete signals).

The computation time that is related to the CS reconstruction is generally considered as a sticking point or, at least, a major limitation due to the fact that is based on complex minimisation algorithms. In the last section, this article presents reconstruction results obtained by using an alternative algorithm based on a fast iterative convolution inpainting [8] for an application to imaging.

II. VARIANCE CALCULATION

To perform the so called adaptive nonuniform sampling, a compact implementation for sub-support variance calculation is required to tune the number of samples to select in it. We consider in this work that one of the main constraint for such a circuit is the efficiency in terms of speed. This specific operation must not imply a conversion of all the samples taken into account in the sub-support. During this variance estimation step, all the samples are sequentially read and provided to the variance calculator structure. However, this operation is interestingly performed during a single conversion time of a first order incremental Σ∆ [9]. In the particular case of such a Σ∆ with an output resolution of N bits and if the digital filter performs an equally weighting (i.e. a N-bit counter), the oversampling ratio is $OSR = 2^N$.

A. 1st order incremental Σ∆ model

The generic structure of a first order incremental Σ∆ is presented in Fig. 2. It is composed of an integrator, a single bit comparator, a feedback loop with a 1-bit DAC and a digital filter which is here a N-bit counter (digital summator). This structure generates a bit stream at the output of the comparator whose the ratio between 1 and 0 is related to the average input which is normalized in comparison with the input range (denoted $[m, M]$). The output of the counter thus provides a digital value representing the input mean. Between each conversion, the Σ∆ is reset to decorrelate successive outputs.

$$y = \frac{OSR}{M-m} \sum_{i=1}^{OSR} x_i - m$$

To simplify the notation, we define the normalized value of $x_i$ and $y$ as $\hat{x_i}$ and $\hat{y}$. This way, thanks to (2), the inputs $\hat{x_i}$ and the output $\hat{y}$ are between 0 and 1.

$$\hat{y} = \frac{y}{OSR} = \frac{1}{OSR} \left\lfloor \frac{OSR}{M-m} \sum_{i=1}^{OSR} \hat{x_i} \right\rfloor$$

B. Variance: Koenig formula

Thanks to the mean operation intrinsically performed by the Σ∆, the easiest way to compute the variance without feedbacks is to take advantage of the Koenig formula (3).

$$\text{Var}(x) = \left( \frac{1}{OSR} \sum_{i=1}^{OSR} \hat{x_i}^2 \right) - \left( \frac{1}{OSR} \sum_{i=1}^{OSR} \hat{x_i} \right)^2$$

Fig. 3 describes the way the variance calculation is performed using two parallel paths each including a Σ∆ converter. Two separate circuits performing a squaring operation are thus required. Indeed, this squaring operation needs to be performed in the analog domain and in the digital one too. Those two different blocks are represented by a $X^2$ in the figure. Notice that, to keep consistency with the top path, the digital subtracter takes only the N most significant bits as input of the bottom path.
**C. Analog squaring circuit**

The analog squaring unit that is considered is based on the schematic presented in [10] (see Fig. 4), and has been optimized to maximize the voltage input range (i.e. from 592mV up to 1.385mV) and the output range when preserving at most the quadratic law over this entire dynamic.

![Fig. 4. Schematic of the analog squaring unit presented in [10]. Optimized parameters are the nmos and pmos transistor sizes: W\textsubscript{n}, L\textsubscript{n}, W\textsubscript{p} and L\textsubscript{p} (in 0.18µm technology process).](image)

This design has the advantage to be relatively compact providing an accurate current output I\textsubscript{out} depending on the squared value of the voltage V\textsubscript{in} with a tunable voltage center V\textsubscript{center} of the squaring function. The output normalization and I to V conversion steps can be performed at the same stage using for instance a transimpedance amplifier. Fig. 7 shows the behaviour of the optimized circuit for a sweep of a normalized input in the acceptable range of inputs.

![Fig. 5. Normalized behaviour of the analog squaring circuit. The maximum error in number of codes for a resolution of 10bits between a square law and the law provided by the analog circuit is of 3 LSBs.](image)

Fig. 7 shows the mismatches between the exact square law function and the function obtained based on the simulation of the analog circuit. In particular, the second derivative of the output in function of a linear input is not flat over the entire range and is tuned a little higher than 2. By minimizing the final maximum output absolute error, the error is lower than 0.3% in absolute value for an input in the acceptable range.

**D. Digital squaring circuit**

To perform the digital squaring operation, we can use either a rearrangement of a traditional digital multiplier as explained in [11] or a combinational floating point squarer [12] if implementation area requirements. Instead of using this kind of designs, the output dataflow of the \(\Sigma\Delta\)'s counter can be exploited. With an appropriate inhibition of a cascaded counter, the squaring operation is performed without complexifying the circuitry. Indeed, thanks to the following equation:

\[
\sum_{k=1}^{y} k = \frac{y(y + 1)}{2},
\]

the computation of \(y^2\) is almost straightforwardly performed. It finally leads to the circuit shown in Fig. 6. To perform the operation described by (4), the 2N-bit counter needs to be inhibited when the comparator output equals 0 meaning that the current output of the N-bit counter is equal to its previous value.

![Fig. 6. Digital squaring unit performing based on a \(\Sigma\Delta\) structure.](image)

Fig. 7 presents the Least Significant Bit (LSB) error of the \(\hat{y}^2\) for a resolution of 10 bits (N = 10).

![Fig. 7. Error in number of codes of the digital squaring circuit for an output resolution of 10 bits.](image)

Finally, for a resolution \(N = 10\), the number of significant bits of the variance is \(N_v = N - 2 = 8\) because, in the worst case, the error related to both the analog and digital squarers respectively represent 3 LSBs and 1 LSB. It thus means that the adaptation of the nonuniform sampling must work if performed according to a variance represented on 8 bits only, removing the two less significant bits. In addition, other mismatches and noises of this order do not impact on the overall system because only an approximation of the variance level is required to adapt the number of samples to consider.
III. ADAPTATION AND NONUNIFORM SAMPLING

The second step of the proposed variance-based nonuniform sampling is the selection of random samples on each sub-support with an adaptive expectancy. This pseudo-random selection is performed using a cellular automaton with a chaotic behaviour generating binary sequences with an adaptive output expectancy. A level of expectancy is thus selected according to non linear 4-level quantization of the digitalized variance.

A. Cellular automaton with a chaotic behavior

Pseudo-random binary values are generated using a cellular automaton following the Wolfram’s rule 30. Indeed, we propose to use a basic 2 neighbours transition function $F_t$ as depicted in Fig. 8 to make it as simple as possible. Alternatively a LFSR could be employed for this purpose but without the advantage to be as scalable and compact as the cellular automaton structure [13].

![Example of an automaton with the 2-neighbours transition function.](image)

**Fig. 8.** Example of an automaton with the 2-neighbours transition function.

B. An expectancy adaptation of the Bernoulli distribution

Multiple combinations of the cell outputs of the cellular automaton are used to provide different expectancies of the final binary outputs. Indeed, Fig. 9 shows a possible arrangement of the cells and AND logic gates. For this specific arrangement, the pseudo-random generator provides as many groups of 4 outputs ($\text{out1, out2, out3, out4}$) as the number of cell columns. In particular, for the proposed automaton with its particular transition rule, the output expectancy is $1/2$. Assuming that the cell outputs are independent identically distributed (iid) random variables, then combining them using AND gates would results in a division of the expectancy by a factor of 2.

![Example of configuration of the cellular automaton with groups of 4 generated bits.](image)

**Fig. 9.** Example of configuration of the cellular automaton with groups of 4 generated bits. The 4 outputs (here: 1,0,0,0 for the first column) represents the outputs with the 4 different expectancies (1/2,1/4,1/8,1/16).

This result shows that the number of selected pixels can efficiently be tuned using the 4 different outputs of the structure presented in Fig. 9. According to the 2-bit quantized variance, the samples that are finally considered for being converted are the ones that are associated to a positive bit output of the pseudo-random generator.

Table I presents the standard deviation of the expectancy of selection for the automaton structure of Fig. 9. This confirms that the distribution of pseudo-random generated values is close to follow the binomial law. Notice that the expectancy for each output is represented by horizontal lines in Fig. 10.

<table>
<thead>
<tr>
<th>Size</th>
<th>4 × 4</th>
<th>8 × 8</th>
<th>16 × 16</th>
<th>32 × 32</th>
</tr>
</thead>
<tbody>
<tr>
<td>out1 (1/2)</td>
<td>0.081/0.125</td>
<td>0.046/0.063</td>
<td>0.016/0.031</td>
<td>0.009/0.016</td>
</tr>
<tr>
<td>out2 (1/4)</td>
<td>0.088/0.108</td>
<td>0.050/0.054</td>
<td>0.024/0.027</td>
<td>0.012/0.014</td>
</tr>
<tr>
<td>out3 (1/8)</td>
<td>0.082/0.083</td>
<td>0.041/0.041</td>
<td>0.020/0.021</td>
<td>0.010/0.010</td>
</tr>
<tr>
<td>out4 (1/16)</td>
<td>0.061/0.063</td>
<td>0.031/0.031</td>
<td>0.016/0.016</td>
<td>0.007/0.008</td>
</tr>
</tbody>
</table>

**TABLE I.** Standard deviation of the expectancy depending on the side size and a specific output. To compare, theoretical results for a binomial distribution are provided in italic.

IV. EXAMPLE OF APPLICATION: IMAGE SAMPLING

A trend in CMOS image sensor [14] is the integration of alternative sensing schemes. For example, compact CS implementations emerge [15]. Recently, adaptive block-based CS [5] [16] is proposed to improve the performances when implementation constraints in anyway involve a block-based approach. On the other hand, the interests of chaotic scanning for image processing purposes is well detailed in [17]. Therefore, our nonuniform sampling scheme has two main advantages for image acquisition. Firstly, its integration at the end-of-column circuitry of a CMOS image sensor is relatively easy thanks to the compactness being compliant with pixel pitch issues. Then, it enables the acquisition of pixel block rows in a parallel fashion and without the need of storing the positions of selected pixel thanks to the deterministic behaviour of the cellular automaton. Finally, it provides CS measurements which are easier to deal with compared to regular measurements. Indeed, instead of using a $l_1$ norm minimization, alternative faster reconstructions can be employed.

Figure 10 presents the resulting expectancy of activation of the proposed pseudo-random generator for different sizes.

![Expectancy estimation of the selection in function of the output of the cellular automaton. Legend marked as 1/2, 1/4, 1/8 and 1/16 respectively represent the outputs out1, out2, out3 and out4.](image)

**Fig. 10.** Expectancy estimation of the selection in function of the output of the cellular automaton. Legend marked as 1/2, 1/4, 1/8, and 1/16, respectively.

![Results in terms of PSNR of the reconstruction depending on the image. Nonuniform sampling is performed with or without variance based adaptation (respectively represented by crosses and circles).](image)

**Fig. 11.** Results in terms of PSNR of the reconstruction depending on the image. Nonuniform sampling is performed with or without variance based adaptation (respectively represented by crosses and circles).
To evaluate the efficiency of the adaptivity of our nonuniform sampling, a fast convolution based reconstruction [8] is used that typically requires less than one second for a 512 × 512 grayscale image. Fig. 11 presents the reconstruction results for 48 grayscale test images (http://decsai.ugr.es/cvg/CG/base.htm) and 3 configurations (cfg) of the adaptivity in terms of expectancy (i.e. 4 quantization levels of the variance). Each point represents a couple of a reconstruction PSNR and a ratio of selected pixels for a particular image. Therefore, each cross is associated to a circle, representing the corresponding control test (i.e. nonuniform equiprobable sampling). Crosses are always above circles. Those three configurations consider signal sub-supports as image blocks of size 4 × 4 where the 2-bit quantization of the variance is defined by 3 threshold levels, allowing the related selection of the random generator output. For cfg 1, an average improvement of the reconstruction PSNR compared of 1.2dB to control 1. Regarding cfg 3, the improvement is of 0.7dB. Due to compressibility variability, those results highly depend on the image. However, it demonstrates that small errors on the variance calculations do not considerably impact on the sampling strategy. The reconstruction PSNR reaches an increase of 2.2dB (cf. Fig. 12), the edges are far better preserved, therefore implying a visible improvement.

Fig. 12. Nonuniform sampling where the ratio between the number of samples and the number of reconstructed pixels is 12%. a) the original image, b) the 2-bit quantized variance, c) nonuniform samples without adaptation, d) nonuniform samples with variance based adaptation of the expectancy, e) and f) the reconstructions respectively based on the samples of c) and d).

V. CONCLUSION

This paper presents a top level implementation for performing nonuniform sampling with a local expectancy adaptation. The design of critical blocks such as the variance calculation over signal sub-supports and the pseudo-random selection are detailed without restrictions on specific sizes to be as generic as possible. Simulations show that the proposed adaptive sensing strategy outperforms a nonadaptive approach.

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