Wavelet coorbit theory in higher dimensions: 
An overview

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Abstract—The continuous wavelet transform is frequently described as a mathematical microscope. In higher dimensions, there is an increasingly larger choice of such microscopes available, which significantly differ in the way that the wavelet (the "lense" of the microscope) is scaled/rotated/sheared etc. by elements of the dilation group. Summarizing recent results, this note presents a unified and comprehensive approach that allows systematic utilization to study many aspects of continuous wavelet transforms. We will show how the dual action can be systematically utilized to study many aspects of continuous wavelet transforms. We will consider the following properties:

- Inversion formulæ and admissibility conditions;
- applicability of coorbit theory, and existence of bandlimited analyzing wavelets;
- simple vanishing moment criteria for wavelet atoms;
- connections to decomposition space theory, and applications to embeddings and dilation invariance for coorbit spaces.

The following sections summarize results from several recent papers [3], [4], [5], [6], [7], [8], in part written jointly with Jonathan Fell, Reihaneh Raisi-Tousi and Felix Voigtlaender. The chief purpose of the present note is to emphasize the pervasive and unifying role of the dual action for the understanding of the various facets of continuous wavelet transforms.

II. INVERSION FORMULÆ AND SQUARE-INTEGRABLE REPRESENTATIONS

The relevant sources for this section are [9], [10], [11], [12], [3]. We call a wavelet \( \psi \) admissible whenever \( \mathcal{W}_\psi \) is an isometry into \( L^2(G) \). In this case, wavelet inversion can be formulated via the weak-sense formula

\[
f = \int_H \int_{\mathbb{R}^d} \mathcal{W}_\psi f(x,h) \pi(x,h) \psi(d(x,h)) \ .
\]

The Plancherel formula provides a characterization of admissible vectors:

\[
\psi \text{ is admissible } \iff \forall_{a,e,\xi} \in \mathbb{R}^d : \int_H |\hat{\psi}(h^T \xi)|^2 dh = 1 \ .
\]

Note that the operator \( \mathcal{W}_\psi : f \mapsto \mathcal{W}_\psi f \) depends on the choice both of the wavelet and the dilation group \( H \). As special cases of this construction that have been studied in more detail, we mention the wavelets based on the similitude group [1], and shearlets [2]. As the dimension increases, the number of possible choices for \( H \) increases dramatically. It is the aim of this note to present a systematic, unified and comprehensive approach towards the study of these transforms and their properties, in particular with a view to understanding the role of the dilation group \( H \). The central feature of \( H \) that will be used throughout is the so-called dual action, which we describe next.

Taking the partial Fourier transform of the wavelet transform with respect to the \( x \) variable results in

\[
(W_\psi(\cdot,h))^\wedge(\xi) = |\det(h)|^{1/2} \hat{f}(\xi)(h^T \xi)
\]

This shows that the CWT can be understood as a continuously labelled filterbank: If the analyzing wavelet \( \psi \) has a Fourier transform that is concentrated inside a set \( U \subset \mathbb{R}^d \), the wavelet transform slice \( \mathcal{W}_\psi f(\cdot,h) \) is a filtered version of the analysed signal \( f \), with frequency content concentrated inside \( h^{-T} U \).

This simple observation establishes the relevance of the dual action \( H \times \mathbb{R}^d \rightarrow \mathbb{R}^d \), defined by \( (h,\xi) \mapsto h^T \xi \).

In this note, we will show how the dual action can be systematically utilized to study many aspects of continuous wavelet transforms.

I. INTRODUCTION

In this note we consider continuous wavelet transforms of multivariate functions, which are defined in a group-theoretic way. For this purpose we fix a closed matrix group \( H \subset \text{GL}(d,\mathbb{R}) \), the so-called dilation group. We let \( G = \mathbb{R}^d \times H \), which is the group of affine mappings generated by \( H \) and all translations. Elements of \( G \) are denoted by pairs \((x,h) \in \mathbb{R}^d \times H \), and the product of two group elements is given by \((x,h)(y,g) = (x + hy, hg) \)\). The left Haar measure of \( G \) is given by \( d(x,h) = |\det(h)|^{-1/2} dx dh \), with \( dh \) denoting left Haar measure on \( H \).

\( G \) acts unitarily on \( L^2(\mathbb{R}^d) \) by the quasi-regular representation defined by

\[
[\pi(x,h)f](y) = |\det(h)|^{-1/2} \cdot f(h^{-1}(y - x)) \ .
\]

Picking a suitable function \( \psi \in L^2(\mathbb{R}^d) \), we then define the continuous wavelet transform of a function \( f \in L^2(\mathbb{R}^d) \) as

\[
\mathcal{W}_\psi f : G \ni (x,h) \mapsto \langle f, \pi(x,h)\psi \rangle \ .
\]

Note that the operator \( \mathcal{W}_\psi : f \mapsto \mathcal{W}_\psi f \) depends on the choice both of the wavelet and the dilation group \( H \). As special cases of this construction that have been studied in more detail, we mention the wavelets based on the similitude group [1], and shearlets [2]. As the dimension increases, the number of possible choices for \( H \) increases dramatically. It is the aim of this note to present a systematic, unified and comprehensive approach towards the study of these transforms and their properties, in particular with a view to understanding the role of the dilation group \( H \). The central feature of \( H \) that
Now the right-hand side has a nice interpretation in terms of the filterbank view obtained after (2): Reconstruction is possible whenever the family of dilated copies of $\psi$ induce a (quadratic) partition of unity on the frequency side.

These observations do not immediately answer when there is a function $\psi$ fulfilling condition (3). However, this can be translated into conditions on the dual action:

**Theorem II.1** ([3]). For a dilation group $H$, there exists an admissible vector $\psi \in L^2(\mathbb{R})$, iff the following conditions hold:

(a) There exists Borel-measurable sets $C \subset U \subset \mathbb{R}^d$, such that $\mathbb{R}^d \setminus U$ has measure zero, $U$ is invariant under $H^T$, every $H^T$-orbit in $U$ intersects $C$ in precisely one point, and for all $\xi \in C$, the dual stabilizer $H_\xi = \{h \in H : h^T \xi = \xi\}$ is compact; and

(b) $H$ is not unimodular.

Of particular importance is the irreducible case [13]; irreducible representations with admissible vectors are called discrete series representations:

**Corollary II.2** ([9], [3]). $\pi$ is a discrete series representation iff the dual action has a unique open dual orbit $\mathcal{O} = H^T \xi$, and the associated stabilizer $H_\xi$ is compact.

III. WAVELET COORBIT SPACES

An important feature of wavelet systems in dimension one is the fact that their approximation-theoretic behaviour is well-understood. Broadly speaking, the class of well-approximated functions with respect to a wavelet ONB coincides with a family of Besov spaces, with the precise range of Besov spaces for which this holds depending on properties of the wavelet, such as compact support, smoothness and vanishing moments. This statement can be understood as a theory of nice wavelets and nice signals: Nice signals are those that can be well approximated by a few elements of a wavelet system consisting of translates and dilates of a nice wavelet. The definition of nice wavelet ensures consistency, which means that the property of being a nice signal is independent of the choice of mother wavelet, as long as the latter comes from the nice wavelet class.

The Besov spaces in higher dimensions have a similar characterization, using either tensor wavelets arising from a wavelet multiresolution analysis, or the continuous wavelet transform associated to the dilation group consisting of scalar dilations combined with rotations. However, it turns out that a consistent theory of nice wavelets and nice signals is available for all dilation groups giving rise to a discrete series representation, using coorbit theory [14], [15]. For this purpose, we introduce weighted mixed $L^p$-spaces on the group $G$, defined by

$$L^p_v,q(G) = \left\{ F : G \to \mathbb{C} : \int_H \left( \int_{\mathbb{R}^d} |F(x,h)|^p v(x,h)^q dx \right)^{q/p} \frac{dh}{\text{det}(h)} < \infty \right\},$$

with the obvious norm $\| \cdot \|_{L^p_v,q}$. Here $1 \leq p,q < \infty$; the definition for $p = \infty$ and/or $q = \infty$ uses the essential supremum instead. The function $v : G \to \mathbb{R}^+$ is a suitably chosen weight function. The idea is to interpret $\|W_\psi f\|_{L^p_v,q}$ as a measure of wavelet coefficient decay. One thus defines good signals $f$ by the property that $W_\psi f \in L^p_v,q(G)$, and introduces the associated coorbit space norm

$$\|f\|_{C^0_{\psi}(L^p_v,q)} = \|W_\psi f\|_{L^p_v,q}.$$

This raises the consistency issue, which is resolved by the results in [14], [15], providing:

- a class $\mathcal{A}$ of nice wavelets with consistent coorbit spaces, i.e., of admissible functions $\psi$ for which the property that $W_\psi f \in L^p_v,q(G)$ is independent of the choice of $\psi \in \mathcal{A}$;
- a class $\mathcal{B} \subset \mathcal{A}$ (of even nicer) wavelets for which the continuous system $\pi(G)\psi$ can be sampled, i.e., replaced by a suitable discrete system $\pi(\Gamma)\psi$, such that mixed $L^p_v,q$-integrability of the continuous transform can be replaced in addition by mixed $L^p_v,q$-summability of the discrete coefficients. Here the class of sampling sets $\Gamma$ is very flexible: Any uniformly discrete and sufficiently dense subset $\Gamma \subset G$ can be used.

Note that the classes $\mathcal{A}, \mathcal{B}$ depend on certain additional weight functions, i.e., $\mathcal{A} = \mathcal{A}_{v_0}, \mathcal{B} = \mathcal{B}_{v_0}$, where $v_0$ can be computed directly from $v, p, q$. One can choose the same $v_0$ for a whole range of values for $p, q$, which allows to define classes of nice wavelets that can simultaneously validate for all associated $L^p_v,q$-spaces.

Coorbit theory, as developed in [14], [15], relies on irreducibility, and although adaptations to the reducible setting are available [16], we will stick to irreducible setup throughout this note. The theory is meaningful whenever $\mathcal{A}$ is nonempty; and this typically imposes stronger conditions on $\pi$ than just irreducibility and square-integrability. Before [4], coorbit theory for the type of continuous wavelet transforms that we consider here had only been established for a select few cases: For the above-mentioned similitude group in [14], for the shearlet group in dimension two in [17], for higher-dimensional shearlet groups in [18]; a similar class of spaces can be defined over symmetric cones [19]. We will see in the next section that it applies to all continuous wavelet transforms arising from square-integrable irreducible representations.

IV. EXISTENCE OF NICE WAVELETS

From now on, we assume that $\pi$ is a discrete series representation. Recall that this entails the existence of a unique open dual orbit $\mathcal{O} \subset \mathbb{R}^d$. We let $\mathcal{O}^c$ denote its complement, hence by definition, $\mathcal{O}^c$ is a closed set of measure zero. We call $\mathcal{O}^c$ the blind spot of the wavelet transform: In the setting we consider here, the value 1 in the admissibility condition (3) is attained for all $\xi \in \mathcal{O}$, whereas this cannot be guaranteed for $\xi \in \mathcal{O}^c$. This can be interpreted as the failure of the wavelet system to resolve frequencies in $\mathcal{O}^c$. We let $C^\infty_c(\mathcal{O})$ denote the space of all smooth functions with support inside $\mathcal{O}$. 
The following theorem shows that the set of (very) nice wavelets is nonempty. Note the role of the open dual orbit in the statement.

**Theorem IV.1** ([4]). Assume that $\pi$ is an irreducible square-integrable representation. Assume that $0 \neq \psi \in L^2(\mathbb{R}^d)$ is such that $\hat{\psi} \in C^\infty_c(O)$. Then $\psi \in B_{v_0}$, for all weights $v_0$ satisfying

$$v_0(x, h) \leq (1 + |x|)^s w(h)$$

with $w$ a submultiplicative, continuous weight on $H$.

Hence coorbit theory is established, with a fairly convenient class of nice wavelets. However, the class of bandlimited Schwartz functions, while easy to write down, is too restrictive for some purposes. In particular, we would like to construct nice wavelets that are compactly supported in space, and what is needed are concrete sufficient criteria for membership in $A$ or $B$ for such functions. This requires considerably more work, and crucially depends on the proper notion of vanishing moments, which involves the blind spot of the dual action.

**Definition IV.2.** Let $r \in \mathbb{N}$ be given. $f \in L^1(\mathbb{R}^d)$ has **vanishing moments in** $O^c$ of **order** $r$ if all distributional derivatives $\partial^\alpha f$ with $|\alpha| \leq r$ are continuous functions, and all derivatives of degree $|\alpha| < r$ are identically vanishing on $O^c$.

In order to derive sufficient vanishing moment conditions for nice wavelets, we next introduce an auxiliary function on the dual orbit, as follows: Fix $\xi \in O$ and let $\text{dist}(\xi, O^c)$ denote the minimal euclidean distance of $\xi$ to $O^c$. Now define

$$A(\xi) = \min \left( \frac{\text{dist}(\xi, O^c)}{1 + \sqrt{\| \xi \|^2 - \text{dist}(\xi, O^c)^2}}, \frac{1}{1 + |\xi|} \right).$$

Briefly, $A$ serves as an envelope function that allows to quantify decay of functions living on $O$ as they approach either infinity or $O^c$. Finally, we pull this function back to the dilation group, by letting

$$A_H(h) = A(h^T \xi).$$

We also need the notion of **Schwartz norms**; for $r, m > 0$, we let

$$|f|_{r,m} = \sup_{x \in \mathbb{R}^d, |\alpha| \leq r} (1 + |x|)^m |\partial^\alpha f(x)|,$$

defined for any function $f : \mathbb{R}^d \to \mathbb{C}$ with suitably many partial derivatives.

With these notations, we can now formulate sufficient conditions for nice wavelets:

**Theorem IV.3** ([7]). Let $v_0$ be a weight satisfying

$$v_0(x, h) \leq (1 + |x|)^s w(h),$$

for a suitable continuous, submultiplicative weight $w$ on $H$. Assume that, for suitable $C_1, C_2, e_1, e_2 \geq 0$, the following estimates hold

$$w(h^{e_1}) A_H(h)^{\epsilon_1} \leq C_1,$$

$$\|h^{e_1}\| A_H(h)^{\epsilon_2} \leq C_2.$$  

Then there exists an integer $t$, explicitly computable from $e_1, e_2, s$ and $d$, such that every nonzero $\psi \in L^2(\mathbb{R}^d)$ with $|\hat{\psi}|_{t,t} < \infty$ and vanishing moments in $O^c$ of order $t$ is contained in $B_{v_0}$.

For more details regarding $t$ and its computation, we refer to [7]. Note that the resulting $t$ may still be fairly large (for the shearlet group in dimension two and relevant weights $v_0$, taking $t = 19$ suffices), but it is the only explicit estimate that we are aware of, even for concrete cases. For compactly supported shearlets, the sufficiency of finitely many vanishing moments is mentioned in [18], but no precise number is given there.

Also, it is important to note that the conditions of the Theorem, technical as they may seem, can be verified in a large variety of cases [7]: E.g., for any (generalized) shearlet group in arbitrary dimension, for any abelian dilation group, and for any dilation group in dimension two.

We next want to address the construction of compactly supported wavelets. Here we will use a higher-dimensional analog of the standard procedure of constructing univariate functions with a prescribed number of vanishing moments: Pick a suitably smooth function $\rho$ and let $\psi = \frac{\partial}{\partial x} \rho$. In higher dimensions, and for wavelets associated to a general dilation group $H$, this procedure once again needs to take in account the structure of the open dual orbit. Here we have:

**Lemma IV.4.** There exists a linear partial differential operator $D$ with constant coefficients and degree $k \leq 2d$ such that, if $f$ and all its partial derivatives of order $\leq sk$ are in $L^1(\mathbb{R}^d)$, then $D^s f$ has vanishing moments of order $s$ in $O^c$.

Note that in concrete cases, both the dual orbit $O$ and the differential operator $D$ can often be computed explicitly. We thus obtain the following simple procedure to construct compactly supported wavelets in $B_{v_0}$:

- Compute $t$ from $v_0$;
- determine $D$;
- choose a compactly supported function $\rho$ with sufficiently many continuous derivatives;
- let $\psi = D^t \rho$.

**V. COORBIT SPACES, DECOMPOSITION SPACES, AND EMBEDDINGS**

The number of suitable dilation groups (and their associated coorbit spaces) grows rapidly with increasing dimension. In order to come to grips with this diversity of groups and spaces, it seems natural to look for systematic ways to classify them. For this purpose, embedding theorems are an important tool. Of particular interest is the situation involving different dilation groups, the prime example in the literature being the embeddings between shearlet coorbit spaces and Besov spaces in [20]. But the question also arises in connection with dilation invariance: The question whether, for a fixed invertible matrix $g$, and given any $f \in L^2(\mathbb{R}^d)$ such that $f \in Co(Y)$ for a certain Banach function space $Y$, the dilate $f \circ g$ is again in $Co(Y)$, is quickly seen to translate into an embedding theorem.
(suitably formulated) of the type $Co_H(Y) \subset Co_H(Y')$. Here $Co_H(Y)$ and $Co_H(Y')$ are wavelet coorbit spaces associated to the dilation groups $H$ and $H' = gHg^{-1}$ respectively, with $Y'$ arising out of $Y$ in a similar fashion. Note that such a statement holds whenever $g \in H$, but for $g \notin H$, the situation is much less clear, and will be seen to depend on $H$.

In this section, we give an informal sketch of a method that allows to systematically tackle these questions, referring to [6] for details. The method relies on viewing coorbit spaces as decomposition spaces, as introduced by Feichtinger and Gröchner [21], and studied further by Borup and Nielsen [22].

The common feature of continuous wavelet transforms on the one hand and decomposition spaces on the other is that of a partition of unity. For the wavelet case, recall that the admissibility condition (3) can be read as a partition of unity. In the decomposition space setting, such a partition is part of the definition: The starting point is an open set $U \subset \mathbb{R}^d$, a covering $Q = (Q_i)_{i \in I}$ of $U$, and a partition of unity subordinate to the covering, i.e., a family $(\varphi_i)_{i \in I}$ of (typically smooth) functions $\varphi_i$ satisfying

$$\text{supp}(\varphi_i) \subset Q_i, \quad \sum_{i \in I} \varphi_i \equiv 1.$$ 

Both the covering and the partition of unity are required to fulfill a list of additional conditions such as local finiteness of the covering (making the sum $\sum_{i \in I} \varphi_i$ pointwise well-defined), norm estimates on the inverse Fourier transform of the $\varphi_i$, etc. The final ingredient is a weight function $u : I \to \mathbb{R}^+$.

We then define the decomposition space with respect to the covering $Q$ and the weight $u$ with integrability exponents $p, q$ as

$$D(Q, L^p, \ell^q_u) := \left\{ f \in D' (U) : \| f \|_{D(Q, L^p, \ell^q_u)} < \infty \right\},$$

where

$$\| f \|_{D(Q, L^p, \ell^q_u)} := \left\| \left( u(i) \cdot \| F^{-1} (\varphi_i f) \|_{L^p(\mathbb{R}^d)} \right)_{i \in I} \right\|_{\ell^q(I)}.$$

To see the analogy to coorbit spaces associated to continuous wavelet transform, one may consider replacing the index set $I$ by $H$, $\varphi_i$ by the Fourier transform of $\pi(0, \hat{h})\psi^*$, where we choose $\psi$ compactly supported in frequency. Then replacing summation by integration against Haar measure essentially recovers the coorbit space norm as the analog of the decomposition space norm of $f$.

But there is a more rigorous way in which coorbit spaces may be viewed as a special case of decomposition spaces, and that uses the notion of induced covering: Let $H$ denote a matrix group with a single open orbit $O$, and associated compact stabilizers. Pick a subset $(h_i)_{i \in I} \subset H$ satisfying two properties: There exist relatively compact neighborhoods $V_1, V_2 \subset H$ of the neutral element such that $h_i V_1 \cap h_j V_1 = \emptyset$ for all $i \neq j$, and in addition $H = \bigcup_{i \in I} h_i V_2$. Pick an open subset $C$ with compact closure in $O$ and such that $O = \bigcup_{i \in I} h_i^{-T} C$ (such a set always exists). Then $Q = (h_i^{-T} C)_{i \in I}$ is an induced covering. With this notion, we get:

**Theorem V.1 ([16], Theorem 43).** Let $1 \leq p, q \leq \infty$, let $v : H \to \mathbb{R}^+$ denote a continuous, submultiplicative weight on $H$, and let $Q = (Q_i)_{i \in I}$ denote an induced covering of the dual orbit $O$. Then there exists an explicitly computable weight $u : I \to \mathbb{R}^+$ such that the Fourier transform induces an isomorphism

$$F : Co(L^p, v) \to D(Q, L^p, \ell^q_u).$$

This theorem can be understood as a partial discretization; essentially, the group $H$ is replaced by the discrete subset $\{ h_i : i \in I \}$. It once again highlights the central role of the dual action: Via the induced covering (and the associated subordinate partition of unity) arising from the dual action, the group specifies a particular decomposition of the frequencies. The theorem states that this decomposition determines the coorbit space. What makes this observation particularly significant is that different dilation groups may have the same (or equivalent) induced coverings.

The theorem provides a common, Fourier-analytic framework that allows to treat all wavelet coorbit spaces, even those arising from different dilation groups, with the same set of tools. It is important to note that the decomposition space setting contains additional, previously studied classes of function spaces: For example, the $\alpha$-modulation spaces, which comprise both the inhomogeneous Besov spaces and the modulation spaces, were introduced as decomposition spaces. But also more recent function spaces, associated to anisotropic wavelet-like systems such as curvelets and cone-adapted shearlets, can be treated with the decomposition space formalism [22], [23].

For a sample application of these observations, let us return to the question of dilation invariance mentioned at the beginning of this section. For details, see Section 9 of [6].

- Let $H = \mathbb{R}^+ \cdot SO(2)$, the group of rotations and scalar dilations. Then the coorbit spaces $Co(L^p, v)$ (i.e., the homogeneous Besov spaces) are dilation invariant under any invertible matrix $g$. Note that this holds even though, generally, $gHg^{-1} \neq H$. Thus different groups may induce the same scale of coorbit spaces. The proof of this fact uses the decomposition space description of coorbit spaces, and a notion of equivalence for coverings.
- If $H$ is the shearlet dilation group, then there exists a weight $v$ on $H$ such $Co(L^p, v)$ is not dilation invariant under a rotation by 90 degrees. This example does not rely on decomposition space theory, but it makes use of the dual action, in particular of the blind spot.

The initial source for embedding theory for decomposition spaces, with focus on $\alpha$-modulation spaces, is Gröchner’s thesis [24]. The upcoming dissertation [25] contains far-reaching sharpenings and extensions of these results, which can be expected to have a considerable impact on the understanding of wavelet coorbit spaces in higher dimensions.

VI. Final Remarks

The main purpose of this note was to highlight the role of the dual action as a unifying concept and the main tool for...
a systematic understanding of the coorbit theory associated to higher-dimensional continuous wavelet transforms.

The usefulness of the dual action is not restricted to the realm of coorbit space theory. As a further application of the ideas sketched in this paper we mention the characterization of singularities using local wavelet coefficient decay. In dimension one, this is well-understood. In higher dimensions, additional challenges arise, mostly because smoothness of a signal becomes a directional feature, as in the definition of the wavefront set. Since the influential discussion of the curvelet resolution of wavefront sets in [26], this has become a kind of benchmark problem for (generalized) wavelet systems, see e.g. [27]. Briefly, the aim is to establish a statement of the following type: Smoothness of a tempered distribution \( u \) near \( x \) in direction \( \xi \) is equivalent to decay of wavelet coefficients \( \mathcal{W}_\psi u(y, h) \) for \( y \) near \( x \), for small-scale wavelets \( \pi(y, h) \psi \) oscillating in directions near \( \xi \). This poses several challenges:

Firstly, to make sense of the notion of small scale, oriented wavelets in the context of a general dilation group. A group like \( H = \mathbb{R}^+ \cdot SO(d) \) allows the easy identification of scale and orientation parameters, but that is the exception.

The second challenge has to do with the notion of angular resolution, that frequently arises in the discussion of wavefront set characterizations [26], [27]: The wavelet system \( \pi(G) \psi \) has to be able to make increasingly fine distinctions between orientations, at least as the scales go to zero.

Once again, the answer to both questions lies in a proper understanding of the dual action of the group. Using this action, one can define a notion of dilations corresponding to small scale, oriented wavelets, and formulate precise conditions ensuring the desired increase in angular resolution as the scales go to zero. As one particular consequence, we obtain an extension of the shearlet characterization of the wavefront set to arbitrary dimensions. We refer to [8] for details.

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