DISTRIB

a GAUSS Library for Statistical Distributions

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written / collected

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The library 'distrib'

Only procedures are listed that are for computing the density, the cumulative distribution function, the inverse of the cumulative distribution function, an to produce random variates. The procedures are all organized in the following manner:

Procedure	First letter of proc	Example
density	D	DBETA
cdf	P	PBETA
inverse of cdf	I	IBETA
random variates	R	RBETA

The main source for distributions are the books:
Johnson, N.L., Kotz, S. and Balakrishnan, N. (1995): Continuous Univariate Distributions, 2nd ed. Volume 1 & 2, New York, Wiley
Johnson, N.L., Kotz, S. and Kemp, A.W. (1992): Univariate Discrete Distributions, 2nd ed., New York, Wiley

Contents

Beta distribution

- **Format**

  `y = DBETA(x,a,b)`
  `z = PBETA(x,a,b)`
  `q = IBETA(p,a,b)`
  `w = RBETA(r,c,a,b)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`b`	scalar, > 0
`p`	(k, m)-matrix, $0 \leq p_{ij} \leq 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{B(a, b)}\, x^{a-1}(1-x)^{b-1} \qquad (0 \leq x \leq 1)$$

- **Remarks**

  Based on GAUSS procedures CDFBETA and RNDBETA.

- **Source**

  `beta.src`

Binomial distribution

- **Format**

  `y = DBINOM(x,p,n)`
  `z = PBINOM(x,p,n)`
  `q = IBINOM(h,p,n)`
  `w = RBINOM(r,c,p,n)`

- **Input**

`x`	(k, m)-matrix
`p`	scalar, $0 \leq p \leq 1$, probability
`n`	integer, ≥ 0
`h`	(k, m)-matrix, $0 \leq h_{ij} \leq 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, h-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; p, n) = \binom{n}{x} p^x (1-p)^{n-x} \qquad (x = 0, 1, 2, \ldots, n)$$

- **Remarks**

  Random numbers are obtained from division of the unit interval based on GAUSS procedure RNDU. Quantiles: $q_h = \arg \min_{0 \leq x \leq n} \mathrm{P}(X \leq x) \geq h$

- **Source**

  `binom.src`

Cauchy distribution

- **Format**

  `y = DCAUCHY(x,a,b)`
  `z = PCAUCHY(x,a,b)`
  `q = ICAUCHY(p,a,b)`
  `w = RCAUCHY(r,c,a,b)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar,
`b`	scalar, > 0
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{\pi b} \cdot \frac{1}{1 + ((x - a)/b)^2}$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `cauchy.src`

Chi distribution

- **Format**

  y = DCHI(x,n)
  z = PCHI(x,n)
  q = ICHI(p,n)
  w = RCHI(r,c,n)

- **Input**

x	(k, m)-matrix
n	scalar, > 0, degrees of freedom
p	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; n) = \frac{1}{2^{(n/2)-1}\Gamma(n/2)} e^{-x^2/2} x^{n-1} \qquad (x > 0)$$

- **Remarks**
  Based on GAUSS procedures CDFCHIC, CDFCHII and RNDGAM.
  Random numbers via square root of χ^2-distributed random variates.

- **Source**

  chi.src

Chisquare distribution

- **Format**

  ```
  y = DCHISQUARE(x,n)
  z = PCHISQUARE(x,n)
  q = ICHISQUARE(p,n)
  w = RCHISQUARE(r,c,n)
  ```

- **Input**

`x`	(k, m)-matrix
`n`	scalar, > 0, degrees of freedom
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; n) = \frac{1}{2^{n/2}\Gamma(n/2)} e^{-x/2} x^{(n/2)-1} \qquad (x > 0)$$

- **Remarks**

  Based on GAUSS procedures CDFCHIC, CDFCHII and RNDGAM.

- **Source**

  `chisq.src`

Empiric distribution

- **Format**

```
 {y,f}  = DEMPIRIC(x)
{y,cf}  = PEMPIRIC(x)
   q    = IEMPIRIC(p,x)
   w    = REMPIRIC(r,c,x,k)
```

- **Input**

`x`	(k, m)-matrix, observed (univariate) values
`p`	(l, n)-matrix, $0 < p_{ij} \leq 1$, probabilities
`r`	integer, > 0, number of rows of observations to be drawn from `x`
`c`	integer, > 0, number of columns of observations to be drawn from `x`
`k`	integer, if `k` equals 1, the sample is done with replacement, otherwise it is done without replacement

- **Output**

`y`	$(h, 1)$-vector, distinct `x`-values
`f`	$(h, 1)$-vector, relativ frequencies of `y`
`cf`	$(h, 1)$-vector, cumulativ relativ frequencies of `y`
`q`	(l, n)-matrix, p-quantiles
`w`	(r, c)-matrix, resampled `x`-values

- **Remarks**

Based on GAUSS procedure RNDU.

Quantiles: $q_p = x_{i:k \cdot m}$ and $i = \arg \min_{1 \leq i \leq k \cdot m} \left(\frac{i}{k \cdot m} \geq p \right)$

- **Source**

`empiric.src`

Exponential distribution

- **Format**

  `y = DEXP(x,a)`
  `z = PEXP(x,a)`
  `q = IEXP(p,a)`
  `w = REXP(r,c,a)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`p`	(k, m)-matrix, $0 \leq p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

  $f(x; a) = a \cdot e^{-a \cdot x} \qquad (x \geq 0)$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `expo.src`

Fisher's F-distribution

- **Format**

  y = DFISH(x,v1,v2)
  z = PFISH(x,v1,v2)
  q = IFISH(p,v1,v2)
  w = RFISH(r,c,v1,v2)

- **Input**

x	(k, m)-matrix
v1	scalar, > 0
v2	scalar, > 0
p	(k, m)-matrix, $0 \leq p_{ij} < 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; v1, v2) = \left(\frac{v_1}{v_2}\right)^{v1/2} \frac{\Gamma\left(\frac{v_1+v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} \frac{x^{(v_1/2)-1}}{(1+\frac{v_1}{v_2}x)^{(v_1+v_2)/2}} \qquad (x > 0)$$

- **Remarks**

  Based on GAUSS procedures CDFFC, CDFNI.
  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `fish.src`

Gamma distribution

- **Format**

  ```
  y = DGAMM(x,a,b)
  z = PGAMM(x,a,b)
  q = IGAMM(p,a,b)
  w = RGAMM(r,c,a,b)
  ```

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`b`	scalar, > 0
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

  $$f(x; a, b) = \frac{x^{a-1}}{b^a \Gamma(a)} e^{-x/b} \qquad (x > 0)$$

- **Remarks**

  Based on GAUSS procedures CDFGAM, GAMMAII and RNDGAM.

- **Source**

  `gamma.src`

Normal (Gauss) distribution

- **Format**

  ```
  y = DGAUSS(x,a,b)
  z = PGAUSS(x,a,b)
  q = IGAUSS(p,a,b)
  w = RGAUSS(r,c,a,b)
  ```

- **Input**

`x`	(k, m)-matrix
`a`	scalar, mean
`b`	scalar, > 0, standard deviation
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{\sqrt{2\pi b^2}} \exp\left[-\frac{1}{2}\left(\frac{x-a}{b}\right)^2\right]$$

- **Remarks**

  Based on GAUSS procedure PDFN, CDFN, CDFNI and RNDN.

- **Source**

  `normal.src`

Standard bivariate normal distribution

- **Format**

  `y = DGAUSS2(xy,r)`
  `z = PGAUSS2(xy,r)`

- **Input**

`xy`	$(k, 2)$-matrix, bivariate data (x_i, y_i)
`r`	scalar, $-1 < r < 1$, correlation between X and Y

- **Output**

`y`	$(k, 1)$-vector, values of the density
`z`	$(k, 1)$-vector, values of the cdf

- **Density**

$$f(x, y; r) = \frac{1}{2\pi\sqrt{1-r^2}} \exp\left[-\frac{1}{2\sqrt{1-r^2}}(x^2 - 2rxy + y^2)\right]$$

- **Remarks**
  Based on GAUSS procedures CDFBVN.
  For random variates see Multivariate Normal Distribution.

- **Source**

  `mgauss.src`

Multivariate normal distribution

- **Format**

  ```
  z = PGAUSSM(x,m,s,r)
  w = RGAUSSM(n,m,s,r)
  ```

- **Input**

`x`	(n, k)-matrix of k-dimensional vectors
`m`	$(k, 1)$-vector, the means
`s`	$(k, 1)$-vector, standard deviations, > 0
`r`	(k, k)-matrix, correlation matrix
`n`	integer, > 0, number of random variates

- **Output**

`z`	$(n, 1)$-vector, values of the cdf
`w`	(n, k)-matrix, k-dimensional random variates

- **Remarks**

  Based on GAUSS procedures CDFMVN and RNDN.

- **Source**

  `mgauss.src`

Generalized error distribution

- **Format**

  `y = DGENERR(x,m,d,phi)`
  `z = PGENERR(x,m,d,phi)`
  `q = IGENERR(p,m,d,phi)`

- **Input**

`x`	(k, m)-matrix
`p`	(k, m)-matrix
`m`	scalar
`d`	scalar, > 0
`phi`	scalar, > 0

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, values of the quantiles

- **Density**

$$f(x; m, d, phi) = \frac{1}{2^{(d/2)+1}\Gamma(\frac{d}{2}+1) \cdot phi} \exp\left[-\frac{1}{2}\left|\frac{x-m}{phi}\right|^{(2/d)}\right]$$

- **Remarks**

  For references see Johnson, Kotz & Balakrishnan (1995): Continuous Distributions, Vol 2. 2nd ed. p. 195
  Based on GAUSS procedures CDFGAM

- **Source**

  `generr.src`

Geometric distribution

- **Format**

  `y = DGEO(x,p)`
  `z = PGEO(x,p)`
  `q = IGEO(h,p)`
  `w = RGEO(r,c,a,b)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`b`	scalar, > 0
`h`	(k, m)-matrix, $0 < h_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, h-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

  $$f(x; p) = p(1-p)^x \qquad (x \in \mathbb{N}_0)$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.
  Quantiles: $q_h = \arg\min_{x \in \mathbb{N}_0} \mathrm{P}(X \leq x) \geq h$

- **Source**

  `geo.src`

Gumbel distribution

- **Format**

  ```
  y = DGUMBEL(x,a,b)
  z = PGUMBEL(x,a,b)
  q = IGUMBEL(p,a,b)
  w = RGUMBEL(r,c,a,b)
  ```

- **Input**

x	(k, m)-matrix
a	scalar
b	scalar, > 0
p	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
r	integer, > 0, number of rows random variates
c	integer, > 0, number of columns random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{b} \exp\left[-\frac{x-a}{b} - \exp\left(\frac{x-a}{b}\right)\right]$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `gumbel.src`

Hypergeometric distribution

- **Format**

  `y = DHYPGEO(x,N,M,k)`
  `z = PHYPGEO(x,N,M,k)`
  `q = IHYPGEO(p,N,M,k)`
  `w = RHYPGEO(r,c,N,M,k)`

- **Input**

`x`	(k, m)-matrix
`N`	integer, > 0, number of 'balls in urn'
`M`	integer, $0 \leq M \leq N$, number of 'marked balls'
`k`	integer, $0 \leq k \leq N$, 'sample size'
`p`	(k, m)-matrix, $0 < p_{ij} \leq 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; N, M, k) = \frac{\binom{M}{x}\binom{N-M}{k-x}}{\binom{N}{k}} \qquad (x \in \{\max(0, k + M - N), \ldots, \min(k, M)\})$$

- **Remarks**

  Random numbers are obtained from division of the unit interval based on GAUSS procedure RNDU. Quantiles: $q_h = \arg\min_x \mathrm{P}(X \leq x) \geq h$

- **Source**

  `hypgeo.src`

Inverse gaussian (wald) distribution

- **Format**

  `y = DINVGAUSS(x,a,b)`
  `z = PINVGAUSS(x,a,b)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`b`	scalar, > 0

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf

- **Density**

  $$f(x; a, b) = \sqrt{\frac{b}{2\pi x^3}} \exp\left(-\frac{b}{2a^2 x}(x-a)^2\right) \qquad (x > 0)$$

- **Remarks**

  Based on GAUSS procedure CDFN.

- **Source**

  `invgauss.src`

Laplace distribution

- **Format**

  `y = DLAPLACE(x,a,b)`
  `z = PLAPLACE(x,a,b)`
  `q = ILAPLACE(p,a,b)`
  `w = RLAPLACE(r,c,a,b)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar
`b`	scalar, > 0
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

  $$f(x; a, b) = \frac{b}{2} e^{-b|x-a|}$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `laplace.src`

Logistic distribution

- **Format**

  ```
  y = DLOGISTIC(x,a,b)
  z = PLOGISTIC(x,a,b)
  q = ILOGISTIC(p,a,b)
  w = RLOGISTIC(r,c,a,b)
  ```

- **Input**

`x`	(k, m)-matrix
`a`	scalar
`b`	scalar, > 0
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer > 0, number of rows random variates
`c`	integer > 0, number of columns random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

  $$f(x; a, b) = \frac{b \cdot e^{-b(x-a)}}{1 + e^{-b(x-a)}}$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `logist.src`

Lognormal distribution

- **Format**

  y = DLOGNORM(x,a,b)
  z = PLOGNORM(x,a,b)
  q = ILOGNORM(p,a,b)
  w = RLOGNORM(r,c,a,b)

- **Input**

x	(k, m)-matrix
a	scalar
b	scalar, > 0
p	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{\sqrt{2\pi} b x} \exp\left(-\frac{1}{2}\frac{(\ln x - a)^2}{b^2}\right) \qquad (x > 0)$$

- **Remarks**

  Based on GAUSS procedures PDFN, CDFN and CDFNI.

- **Source**

  lognorm.src

Maxwell distribution

- **Format**

  y = DMAXWELL(x,s)
  z = PMAXWELL(x,s)
  q = IMAXWELL(p,s)
  w = RMAXWELL(r,c,s)

- **Input**

x	(k, m)-matrix
s	scalar, > 0
p	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; n) = \sqrt{\frac{2}{\pi}} \frac{x^2}{s^{1.5}} \exp\left(-\frac{x^2}{2s}\right) \qquad (x \geq 0)$$

- **Remarks**

  Based on GAUSS procedure GAMMAII.
  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  chi.src

Multinomial distribution

- **Format**

  `y = DMULTI(x,p,n)`
  `z = VMULTI(x,p,n)`
  `w = RMULTI(m,p,n)`

- **Input**

`x`	(l,k)-matrix of integer, $0 \leq x_i \leq n$
`p`	$(k,1)$-vector, $\sum p_i = 1$, probabilities
`n`	integer, $n \geq 0$
`m`	integer, number of k-dimensional random variates

- **Output**

`y`	$(l,1)$-vector, values of the density
`z`	$(l,1)$-vector, values of the cdf
`w`	(m,k)-matrix, k-dimensional random variates

- **Density**

$$f(x_1,\ldots,x_k) = \frac{n!}{x_1!x_2!\ldots x_k!} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k} \quad \text{with } x_i \geq 0 \text{ and } \sum x_i = n$$

$$F(x_1,\ldots,x_k) = \sum_j f(y_1^j,\ldots,y_k^j) \quad \text{with } \mathbf{y}^j \in \{\mathbf{y} \,|\, y_i \leq x_i,\ \sum y_i^j = n\} \subset \mathbb{N}^k$$

- **Source**

  `multinom.src`

Multivariate normal distribution

- **Format**

  `z = PGAUSSM(x,m,s,r)`
  `w = RGAUSSM(n,m,s,r)`

- **Input**

`x`	(n, k)-matrix of k-dimensional vectors
`m`	$(k, 1)$-vector, the means
`s`	$(k, 1)$-vector, standard deviations, > 0
`r`	(k, k)-matrix, correlation matrix
`n`	integer, > 0, number of random variates

- **Output**

`z`	$(n, 1)$-vector, values of the cdf
`w`	(n, k)-matrix, k-dimensional random variates

- **Remarks**

  Based on GAUSS procedures CDFMVN and RNDN.

- **Source**

  `mgauss.src`

Negative binomial distribution

- **Format**

  `y = DNEGBINOM(x,p,k)`
  `z = PNEGBINOM(x,p,k)`
  `q = INEGBINOM(h,p,k)`
  `w = RNEGBINOM(r,c,p,k)`

- **Input**

`x`	(n, m)-matrix
`p`	scalar, $0 < p < 1$, probability
`k`	scalar, > 0
`h`	(n, m)-matrix, $0 < h_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(n, m)-matrix, values of the density
`z`	(n, m)-matrix, values of the cdf
`q`	(n, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; p, k) = \frac{\Gamma(x+k)}{\Gamma(k)\Gamma(x+1)} p^x (1-p)^k \qquad (x \in \mathbb{N}_0)$$

- **Remarks**

  Based on GAUSS procedures CDFCHIC and RNDNB.
  Quantiles: $q_h = \arg\min_{x \in \mathbb{N}_0} \mathrm{P}(X \leq x) \geq h$

- **Source**

  `negbinom.src`

Normal distribution

- **Format**

  y = DGAUSS(x,a,b)
  z = PGAUSS(x,a,b)
  q = IGAUSS(p,a,b)
  w = RGAUSS(r,c,a,b)

- **Input**

x	(k, m)-matrix
a	scalar, mean
b	scalar, > 0, standard deviation
p	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{\sqrt{2\pi b^2}} \exp\left[-\frac{1}{2}\left(\frac{x-a}{b}\right)^2\right]$$

- **Remarks**

  Based on GAUSS procedure PDFN, CDFN, CDFNI and RNDN.

- **Source**

  normal.src

Pareto distribution

- **Format**

  ```
  y = DPARETO(x,a,k)
  z = PPARETO(x,a,k)
  q = IPARETO(p,a,k)
  w = RPARETO(r,c,a,k)
  ```

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`k`	scalar, > 0
`p`	(k, m)-matrix, $0 \leq p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

  $$f(x; a, k) = ak^a x^{-(a+1)} \qquad (x \geq k)$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `pareto.src`

Poisson distribution

- **Format**

  `y = DPOISSON(x,l)`
  `z = PPOISSON(x,l)`
  `q = IPOISSON(p,l)`
  `w = RPOISSON(r,c,l)`

- **Input**

`x`	(k, m)-matrix
`l`	scalar, > 0
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; \lambda) = e^{-\lambda} \frac{\lambda^x}{x!} \qquad (x \in \mathbb{N}_0)$$

- **Remarks**

  Based on GAUSS procedure RNDP.

- **Source**

  `poisson.src`

Polya distribution

- **Format**

  `y = DPOLYA(x,n,p,a)`
  `z = PPOLYA(x,n,p,a)`

- **Input**

x	(k, m)-matrix
n	integer, > 0
p	scalar, $0 < p < 1$, probability
a	scalar, > 0

- **Output**

y	(k, m)-matrix, values of density
z	(k, m)-matrix, values of cdf

- **Density**

$$f(x; n, p, a) = \binom{n}{x} \frac{\prod_{i=1}^{x} [b + (i-1)a] \prod_{i=1}^{n-x} [((1-p) + (i-1)a]}{\prod_{i=1}^{n} [1 + (i-1)a]}$$

- **Reference**

  M.Fisz (1978): Wahrscheinlichkeitsrechnung und mathematische Statistik, Deutscher Verlag der Wissenschaften

- **Source**

  `polya.src`

Power distribution

- **Format**

  y = DPOWER(x,a,b)
  z = PPOWER(x,a,b)
  q = IPOWER(p,a,b)
  w = RPOWER(r,c,a,b)

- **Input**

x	(k, m)-matrix
a	scalar, > 0
b	scalar, > 0
p	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{a}{b}\left(\frac{x}{b}\right)^{a-1} \qquad (0 < x \leq b)$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  power.src

Rayleigh distribution

- **Format**

  `y = DRAY(x,s)`
  `z = PRAY(x,s)`
  `q = IRAY(p,s)`
  `w = RRAY(r,c,s)`

- **Input**

`x`	(k,m)-matrix
`s`	scalar, > 0
`p`	(k,m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k,m)-matrix, values of the density
`z`	(k,m)-matrix, values of the cdf
`q`	(k,m)-matrix, p-quantiles
`w`	(r,c)-matrix, random variates

- **Density**

$$f(x;n) = \frac{x}{s^2} \exp\left(-\frac{x^2}{2s^2}\right) \qquad (x \geq 0)$$

- **Remarks**

  χ_2-Distribution for $s = 1$.
  Inverse cdf is based on GAUSS procedure GAMMAII.
  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `chi.src`

Ratio of Quadratic Forms

- **Format**

  `z = PRQUADFORM(x,a,b)`
  `q = IRQUADFORM(p,a,b)`
  `w = RRQUADFORM(r,a,b)`

- **Input**

`x`	scalar, $(x > 0)$
`a`	(n, p)-matrix, symmetric
`b`	(n, p)-matrix, symmetric and positive definite
`p`	scalar, $0 < p < 1$, probability
`r`	integer, > 0, number random variates

- **Output**

`z`	scalar, value of the cdf
`q`	scalar, p-quantile
`w`	$(r, 1)$-vector, random variates

- **Density**

  Density is not availabe. The cdf is
  $F(x; \mathbf{A}, \mathbf{B}) = P(\mathbf{u}'\mathbf{A}\mathbf{u} / \mathbf{u}'\mathbf{B}\mathbf{u} \leq x)$ where $\mathbf{u} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

- **Remarks**

  Programs were written by Anurag N Banerjee as freeware for public non-commercial use.
  PRQUADFORM is based on numerical integration. Random numbers based on GAUSS procedure RNDU.

- **Reference**

  Abrahamse, A.P.J and Koerts,J. (1969): On the Theory and application of the General Linear Model , Rotterdam University Press.
  Imhof, J.P (1961): Computing the Distribution of Quadratic forms of Normal Variables; Biometrika 48, 419-426.

- **Source**

  `quadform.src`

Student t-distribution

- **Format**

  `y = DSTUDENT(x,v)`
  `z = PSTUDENT(x,v)`
  `q = ISTUDENT(p,v)`
  `w = RSTUDENT(r,c,v)`

- **Input**

`x`	(k, m)-matrix
`v`	scalar, > 0, degrees of freedom
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows for random variates
`c`	integer, > 0, number of columns for random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; v) = \frac{\Gamma((v+1)/2)}{\sqrt{\pi v}\,\Gamma(v/2)} \left(1 + \frac{x^2}{v}\right)^{-(v+1)/2}$$

- **Remarks**
  Based on GAUSS procedure CDFTC and CDFTCI. Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `student.src`

Symmetric Stable distribution

- **Format**

```
    y      =DSYMSTAB(x,a)
 {y,dy}    =DDSYMSTAB(x,a)
{z,y,dy}   =PSYMSTAB(x,a)
    w      =RSYMSTAB1(r,c,a)
    w      =RSYMSTAB2(r,c,a,b)
```

- **Input**

`x`	$(k, 1)$ vector
`a`	scalar, $0 < a \leq 2$; for `RSYMSTAB`: $0.1 \leq a \leq 2$
`b`	scalar, > 0, shape parameter; for `RSYMSTAB`: $-1 \leq b \leq 1$
`r`	scalar, integer ¿0, number of rows of matrix of random variates
`c`	scalar, integer ¿0, number of columns of matrix of random variates

- **Output**

`y`	$(k, 1)$-vector, values of the density
`dy`	$(k, 1)$-vector, values of the derivative of the density
`z`	$(k, 1)$-vector, values of the cdf
`w`	(r, c)-vector of random variates

- **Density**

  Densities are not avaiable. The characteristic function is:

$$\mathrm{E}\left(e^{iXt}\right) = \begin{cases} \exp\left[-|t|^a(1 - b \cdot \operatorname{sign}(t)\tan(\pi a/2)\right] & a \neq 1 \\ \exp\left[-|t|(1 - b \cdot \frac{2}{\pi}\operatorname{sign}(t)\ln(|a|)\right] & a = 1 \end{cases}$$

  If $a < 0.1$, probability of overflow in `RSYMSTAB` becomes non-negligible.

- **Remarks**

  Programs were written by J.H.McCulloch, June 1993. The code is written and submitted for public, non-commercial use. See the source for further hints.

- **Source**

  `symstab.src`

Uniform distribution

- **Format**

  y = DUNIFORM(x,a,b)
  z = PUNIFORM(x,a,b)
  q = IUNIFORM(p,a,b)
  w = RUNIFORM(r,c,a,b)

- **Input**

x	(k, m)-matrix
a	scalar, lower bound of support
b	scalar, upper bound of support
p	(k, m)-matrix, $0 \leq p_{ij} \leq 1$, probabilities
r	integer, > 0, number of rows of matrix of random variates
c	integer, > 0, number of columns of matrix of random variates

- **Output**

y	(k, m)-matrix, values of the density
z	(k, m)-matrix, values of the cdf
q	(k, m)-matrix, p-quantiles
w	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{b-a} \qquad (a \leq x \leq b)$$

- **Remarks**

  Based on GAUSS procedure RNDU.
  Quantiles: $q_0 = \min(a, b),\ q_1 = \max(a, b)$.

- **Source**

  uniform.src

von Mises Distribution

- **Format**

  `y = DVMISES(x,a,b)`
  `w = RVMISES(r,c,a,b)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar, $-\pi \leq a \leq \pi$, location parameter
`b`	scalar, > 0, scale parameter
`r`	integer, > 0, number of rows of matrix of random variates
`c`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, c)-matrix, random variates

- **Density**

$$f(x; a, b) = \frac{1}{2\pi I_0(b)} \exp[b \cos(x - a)] \qquad (x \in [-\pi, \pi])$$

- **Remarks**

  This is a distribution on the circle. $I_0(b)$ is the Bessel-function of the first kind. Random numbers based on GAUSS procedure RNDVM.

- **Source**

  `vmises.src`

Weibull distribution

- **Format**

  `y = DWEIBULL(x,a,c)`
  `z = PWEIBULL(x,a,c)`
  `q = IWEIBULL(p,a,c)`
  `w = RWEIBULL(r,co,a,c)`

- **Input**

`x`	(k, m)-matrix
`a`	scalar, > 0
`c`	scalar, > 0
`p`	(k, m)-matrix, $0 < p_{ij} < 1$, probabilities
`r`	integer, > 0, number of rows of matrix of random variates
`co`	integer, > 0, number of columns of matrix of random variates

- **Output**

`y`	(k, m)-matrix, values of the density
`z`	(k, m)-matrix, values of the cdf
`q`	(k, m)-matrix, p-quantiles
`w`	(r, co)-matrix, random variates

- **Density**

$$f(x; a, c) = \frac{c}{a}\left(\frac{x}{a}\right)^{c-1} \exp\left[-\left(\frac{x}{a}\right)^{c}\right] \qquad (x > 0)$$

- **Remarks**

  Random numbers by inversion method based on GAUSS procedure RNDU.

- **Source**

  `weibull.src`